Possibility of capturing a meteorite at L4 Point

Motivation: : In this simulation, I want to explore the possibility of capturing the third celestial object at L4 Lagrangian Point in Sun-Jupiter system.

1 Effective Potential Field

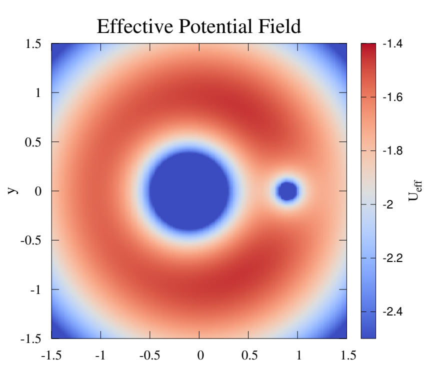
In a binary system, the total potential field is the combination of each star or planet's gravitational potential field. And in rotating frame of reference, centrifugal potential energy should also be included:

$$U = -\frac{Gm_s}{r_s} - \frac{Gm_p}{r_p} - \frac{1}{2}(r\omega)^2 \tag{1}$$

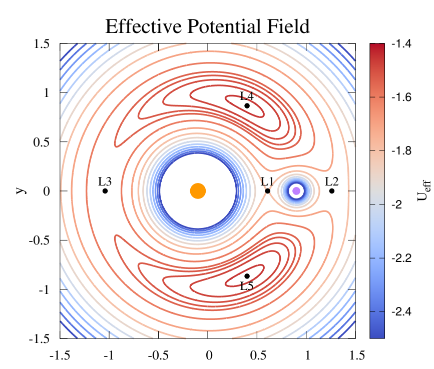
where m_s is the mass of star, m_p is the mass of planet, ω is the angular frequency of the rotating frame, r_s, r_p and r are the distance to the star, planet and the center. To simplify this equation, we rewrite the r terms and m terms using nondimensional variables, $r_s = \rho_s R$, $r_p = \rho_p R$, $r = \rho R$, $M = m_s + m_p$, $m_s = M\mu$ and $m_p = M(1-\mu)$,then the equation becomes:

$$U = \frac{GM}{R}\left[-\frac{1-\mu}{\rho_s} - \frac{\mu}{\rho_p} - \frac{1}{2}\rho^2\right] = \frac{GM}{R}u \tag{2}$$

where ρ_s, ρ_p and ρ are dimensionless distance to star, planet and the center of the binary system. And the following figures show the effective potential field u.



(a) Heatmap of Effective Potential Field



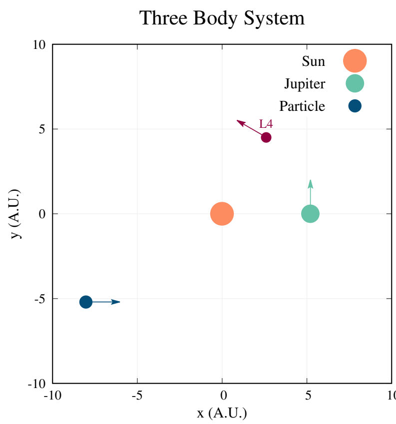
(b) Contour of Effective Potential Field

Figure 1: Effective Potential. (In this figure, I set the mass ratio $\mu = 0.1$ for better demonstration. In the later simulation, I use the real mass and distance value for Sun and Jupiter.)

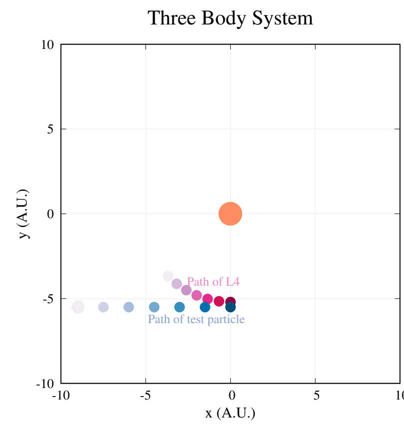
2 Simulation of the Capture

2.1 Imagined Results

I design the process of the capture of the third particle. At first, the test particle is not located in the binary system. With the evolution of time, the position of the test particle coincides with the L4 Lagrangian point, and then the test particle is located at the L4 point, revolving around the Sun in circular orbit like Jupiter. This process could be seen in Fig.(2). Since the total energy is conservative in the whole process, the test particle must have the same energy as particles in Jupiter Trojan belt.



(a) Initial State of the Simulation



(b) Capture of the Celestial Object

Figure 2: The initial and final process of capture. For (b) figure, colors from light to dark represent evolution over time

Since it takes $\frac{5}{6}$ of the Jupiter period time for L4 point to move directly below the Sun, I set the initial velocity of the test particle so that it also moves to that position using this time. Then the initial state of position and velocity can be gained by solving the following equation, (also see Fig.(3)):

$$E_{tot} = -\frac{Gm_s}{r_s} + \frac{1}{2}(r_p\omega)^2 = -\frac{Gm_s}{r} + \frac{1}{2}v^2 \tag{3}$$

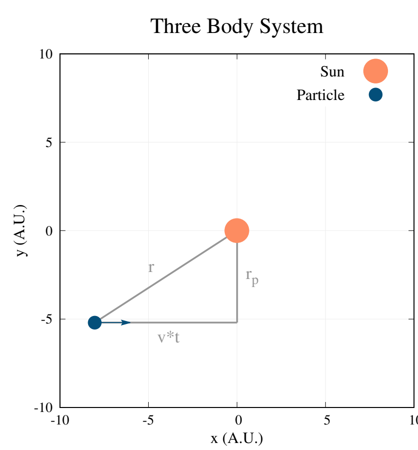


Figure 3: Initial State

The initial conditions are shown in Table(1).

	Sun	Jupiter	Test Particle
mass(m_{unit})	1.00	9.55×10^{-4}	5.03×10^{-28}
position $(A.U.)$	$(-4.97 \times 10^{-3}, 0, 0)$	$(5.20, 0, 0)$	$(-8.03, -5.20, 0)$
velocity $(A.U./year)$	$(0, -2.63 \times 10^{-3}, 0)$	$(0, 2.75, 0)$	$(0.81, 0, 0)$

Table 1: Initial Conditions for Three-Body Simulation

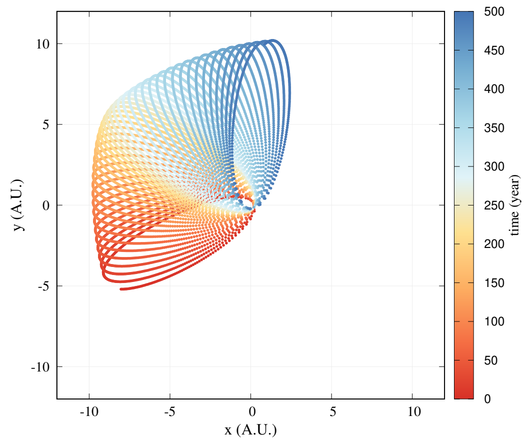
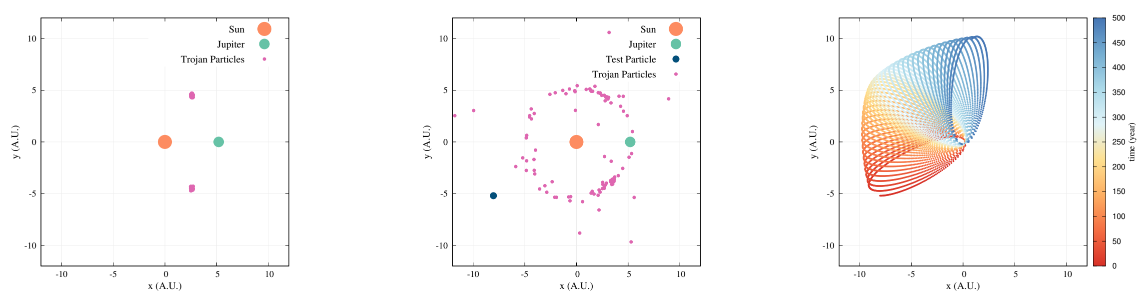


Figure 4: Path of Test Particle in Three-Body Simulation

2.2 Simulated Result

Fig.(4) shows the result of the motion of the test particle. This test particle has a large deflection before reaching the preset point, and it does not move according to the expected path, and although it has been rotating around the Sun, it does not move around the Sun like particles at point L4 in circular motion.

Then, I set the Trojan belt particles to test whether these particles could help capture. In Fig.(5c) shows the path of test particle with the existence of Trojan belt particles. Comparing Fig.(5c) and (4), we could see that there is almost no difference regardless of whether the Trojan belt exists or not.



(a) Initial State of Trojan Belts **(b)** Adding Test Particle **(c)** Path of Test Particle

Figure 5: The initial conditions of the Trojan particles are set as follows: on the basis of the point L4 or L5 having a circular motion around the Sun, a 0-5% uniformly distributed perturbation is added to the velocity and position of each particle. The mass of each Trojan belt particle is 10^{-6} of Earth's mass and there are 100 Trojan particles. When the Trojan belt forms, test particle is added.

3 Analysis

This result could be explain by the effective potential and initial condition. For effective potential, it could be gained by:

$$E_{tot} = \frac{1}{2}m\dot{r}^2 + \frac{1}{2}m\left(r\dot{\theta}\right)^2 - \frac{GMm}{r} \tag{4}$$

$$= \frac{1}{2}m\dot{r}^2 + \frac{1}{2}m\left(r\frac{L}{mr}\right)^2 - \frac{GMm}{r} \tag{5}$$

$$= \frac{1}{2}m\dot{r}^2 + \frac{L^2}{mr^2} - \frac{GMm}{r} = \frac{1}{2}m\dot{r}^2 + U_{eff} \tag{6}$$

where L is the angular momentum and it is conservative in the whole process. For a given initial total energy, we could have different angular momentum corresponding to the same total energy. So, different initial angular momentum would lead to different effective potential lines, in Fig.(6). And we can see that circular motion is possible only when the minimum effective potential energy is equal to the total energy, which means there is no radial motion. When the total energy does not equal to minimum effective potential, the particle's distance from the Sun will vary. And in my simulation, the initial condition of the test particle contains the radial velocity term, that is, the radial velocity is not zero, so it is not possible for the test particle to orbit around the Sun with circular path just like the motion of Jupiter.

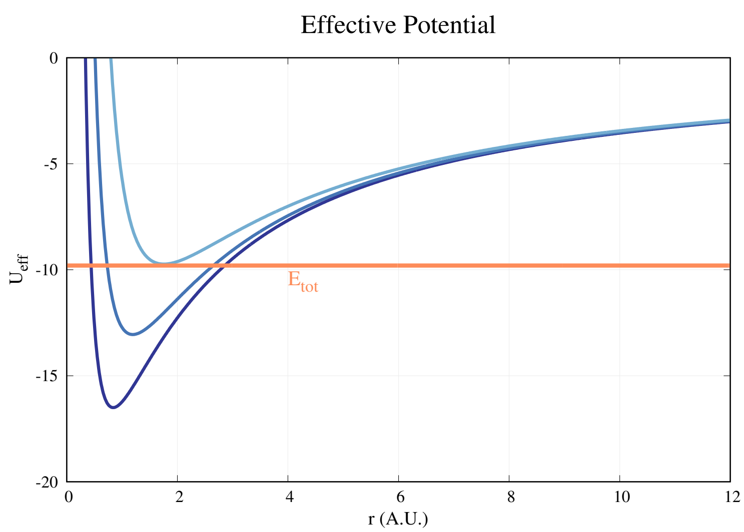


Figure 6: Effective Potential Lines with Different Angular Momentum